A comparison of the series and parallel Masing-Iwan model in 2D

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Studentship acknowledgement: PISA Project
NUMGE funding acknowledgement: Cathie Associates
Motivation

- Wind and wave loading on wind turbines can come from different directions
- Plastic soil-reaction (p-y) curves in 2D
- Small-scale experimental tests for monopiles in sand under (limited) cyclic loading show Masing-behaviour (Abadie, 2015)
Masing rules

1. Initial tangent stiffness = reversal stiffness
2. Unload & reload shape = backbone x 2
3. Re-join backbone after max. previous load
4. Follow previous curve after intersection with unload-reload curve
Kinematic hardening model

Parallel

Series

Diagram showing parallel and series configurations of elements with stress (σ) and strain (ε) relationships.
Hyperplasticity

- Framework guarantees thermodynamic acceptability
- Material model captured in 2 scalar functions
- Facilitates comparison of competing models and development of new models

\[ \bar{X}_{ij} = X_{ij} \]
Hierarchy of models

1D: \( \chi^2 \leq k^2 \)

2D: \( \chi_x^2 + \chi_y^2 \leq k^2 \)

Continuum: \( \chi_{ij}' \chi_{ij}' \leq 2k^2 \)
Model comparison

### Series

1D

\[ g = -\frac{\sigma^2}{2H_0} - \sigma \sum_{i=1}^{n_s} \alpha_i + \sum_{i=1}^{n_s-1} \frac{H_i}{2} \alpha_i^2 \]

\[ d = \sum_{i}^{n_s} k_i \sqrt{\hat{\alpha}_i^2} \]

2D

\[ g = -\frac{\sigma_x^2 + \sigma_y^2}{2H_0} - \sigma_x \sum_{i=1}^{n_s} \alpha_{ix} - \sigma_y \sum_{i=1}^{n_s} \alpha_{iy} \]

\[ + \sum_{i=1}^{n_s-1} \frac{H_i}{2} \left( \alpha_{ix}^2 + \alpha_{iy}^2 \right) \]

\[ d = \sum_{i}^{n_s} k_i \sqrt{\hat{\alpha}_{ix}^2 + \hat{\alpha}_{iy}^2} \]

### Parallel

1D

\[ g = -\frac{(\sigma + \sum_{i=1}^{n_s} H_i \alpha_i)^2}{2E_0} + \sum_{i=1}^{n_s} \frac{H_i}{2} \alpha_i^2 \]

\[ d = \sum_{i}^{n_s} k_i \sqrt{\hat{\alpha}_i^2} \]

2D

\[ g = -\frac{(\sigma_x + \sum_{i=1}^{n_s} H_i \alpha_{ix})^2}{2E_0} - \frac{(\sigma_y + \sum_{i=1}^{n_s} H_i \alpha_{iy})^2}{2E_0} \]

\[ + \sum_{i=1}^{n_s} \frac{H_i}{2} \left( \alpha_{ix}^2 + \alpha_{iy}^2 \right) \]

\[ d = \sum_{i}^{n_s} k_i \sqrt{\hat{\alpha}_{ix}^2 + \hat{\alpha}_{iy}^2} \]
Model comparison

Parallel

Series
Code available on GitHub

github.com/WBeuckelaers/hyperplasticity